Georgia High School Graduation Tests

Test Content Descriptions for 2011

Based on Georgia Performance Standards

Mathematics

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Introduction

The Georgia State Board of Education, in 2005, approved Georgia Performance Standards (GPS) in mathematics for use in all Georgia public schools. The GPS curriculum was implemented in grades nine through 12 in 2008-2009. Beginning in spring 2011, first-time 11th grade test takers will take a mathematics graduation test based solely on the GPS.

The GPS standards will be assessed with cognitively rich items reflecting the range and depth of knowledge inherent in the GPS. A new cut score (passing) and a new scale score will be set for the GPS version of the Mathematics Georgia High School Graduation Tests and applied to the spring 2011 test.

Program Purpose

The Georgia High School Graduation Tests (GHSGT) measure whether Georgia High School students have mastered essential concepts and skills from the state-adopted curriculum deemed necessary to earn a diploma. Students earning a regular Georgia diploma must pass all four parts of the GHSGT and the Georgia High School Writing Assessment in addition to meeting other local and state graduation requirements.

GHSGT Content Descriptions

The GHSGT Content Descriptions are provided to acquaint Georgia educators with the content coverage of the GHSGT. Only the knowledge, concepts, and skills addressed in the GPS provided in this document are assessed on the GHSGT. Committees of Georgia educators reviewed the curriculum and provided guidance for this assessment program.

It is important to note that some curricular standards are better suited for classroom or individual assessment rather than large-scale, paper-and-pencil assessment. While those curricular standards designed for classroom/individual assessment are not included in the Content Descriptions, the knowledge, concepts, and skills outlined are often required for the mastery of the standards that are assessed. Therefore, the GHSGT Content Descriptions are in *no way* intended to substitute for the GPS; they are provided to help educators better understand how the curriculum will be assessed. Further, the GHSGT Content Descriptions *by no means* suggest *when* concepts and skills should be introduced in the instructional sequence; rather, their purpose is to communicate when concepts and skills will be assessed on the GHSGT. Georgia law requires educators to teach the standards set forth in the state-adopted curriculum (i.e., the GPS). The GPS are located at http://www.georgiastandards.org.

Mathematics Domains

To provide reliable measures of student achievement and to give structure to the assessment program, the content standards contained in the GPS were grouped into content domains. Each domain was created by combining standards that share similar content characteristics. Three domains were identified for Mathematics.

- **Algebra**
  Students will demonstrate the ability to explore functions; solve radical, simple quadratic and rational equations; simplify and perform operations with radical, polynomial, and rational expressions. Students will investigate piecewise, exponential, and quadratic functions using numerical, analytical, and graphical approaches, focusing on the use of these functions in problem-solving situations; solve equations and inequalities related to these functions; explore the inverses of functions; use the complex number system.

- **Geometry**
  Students will demonstrate the ability to explore, understand, and use the formal language of reasoning and justification in both algebraic and geometric contexts; apply properties of polygons; and determine distances and points of concurrence. Students will understand and apply properties of right triangles and right-triangle trigonometry; understand and apply properties of circles and spheres, and use them in determining related measures.

- **Data Analysis**
  Students will demonstrate the ability to determine probability; use both permutations and combinations to find the number of outcomes; pose questions to be answered by collecting data; and organize, represent, investigate, interpret, and make inferences from data. Students will demonstrate understanding of data analysis by posing questions to be answered by collecting data; organize, represent, investigate, interpret, and make inferences from data; compare data for two different samples and/or populations using measures of central tendency and measures of spread, including standard deviation; use linear and quadratic regressions to analyze data and to make inferences.

Process Standards

The GPS in mathematics require content to be taught in conjunction with process skills identified as the process standards. These process standards are necessary for students to master each of the mathematics content standards. Problem solving, reasoning, representation, connections, and communication are the critical dimensions of mathematical proficiency that all students need.

The concepts and skills inherent in the process standards are integrated in items across the three content domains.

Overview of the Process Standards

- Students will solve problems (using appropriate technology).
- Students will reason and evaluate mathematical arguments.
- Students will communicate mathematically.
- Students will make connections among mathematical ideas and with other disciplines.
- Students will represent mathematics in multiple ways.

Associated GPS

MM1P1 through MM1P5 within content from MM1A1 through MM1D3
MM2P1 through MM2P5 within content from MM2A1 through MM2D2
MM3P1 through MM3P5 within content from MM3A2 through MM3G1

Associated GPS Concepts and Skills

- Building new mathematical knowledge through problem solving.
- Solving problems that arise in mathematics and in other contexts.
- Applying and adapting a variety of appropriate strategies to solve problems.
- Reflecting on and monitoring the process of mathematical problem solving.
- Recognizing reasoning and proof as fundamental aspects of mathematics.
- Making and investigating mathematical conjectures.
- Developing and evaluating mathematical arguments and proofs.
- Selecting and using various types of reasoning and methods of proof.
- Organizing and consolidating mathematical thinking through communication.
- Communicating mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyzing and evaluating mathematical thinking and strategies of others.
- Using the language of mathematics to precisely express mathematical ideas.
- Recognizing and using connections among mathematical ideas.
- Understanding how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognizing and applying mathematics in contexts outside of mathematics.
- Creating and using representations to organize, record, and communicate mathematical ideas.
- Selecting, applying, and translating mathematical representations to solve problems.
- Using representations to model and interpret physical, social, and mathematical phenomena.

Mathematics

Domain: Algebra (approximately 36% of the test)

Overview of the Domain

- Students will use graphs, tables, and simple algebraic techniques to explore and interpret the characteristics of functions.
- Students will simplify and perform operations with radical expressions, polynomials, and rational expressions.
- Students will solve radical, quadratic, and rational equations.
- Students will investigate step and piecewise functions, including greatest integer and absolute value functions.
- Students will explore exponential functions.
- Students will analyze quadratic functions in the forms $f(x) = ax^2 + bx + c$ and $f(x) = a(x - h)^2 + k$.
- Students will solve quadratic equations and inequalities in one variable.

Associated GPS Standards

MM2N1a,b,c	MM1A1b,c,d,e,f,g,h,i	MM1A2a,b,c,e,f	MM1A3a,b,c,d	
MM2A1b,c	MM2A2a,b,c,d,f,g	MM2A3a,b,c	MM2A4b	MM2A5b
MM3A2b,e	MM3A5a			

Associated GPS Concepts and Skills

Assessment of this domain will focus on student ability to

- graph and identify graphs of basic functions (limited to $f(x) = x^n$, where $n = 1$ to 3, $f(x) = |x|$, $f(x) = \sqrt{x}$ and $f(x) = \frac{1}{x}$);
  - select a graph that matches a particular function;
  - select a function that matches a given graph; and
  - understand that graphs are geometric representations of functions.

- graph transformations of basic functions;
  - examine and identify shifts, stretches, and shrinks of parent functions; and
  - explore and identify reflections across the *x*- and *y*-axes of parent functions.

- investigate and explain the characteristics of quadratic, cubic, inverse, absolute value, and square root functions (using linear functions only as a building block);
  - identify a domain (the set of inputs) and a range (the set of outputs);
  - understand set notation;
  - explore the zeros/solutions;
  - find *x*- and *y*-intercepts;
  - determine intervals of increase and decrease;
  - locate maximum and minimum values; and
  - explain end behavior.
- relate the characteristics of a function to a given context;

  - utilize graphs, tables, and words to explain and predict the behavior of a function; and
  - understand the distinctions between discrete and continuous domains.
- recognize sequences as functions with domains that are whole numbers greater than zero;
  - examine sequences given in tables, algebraically, or by producing a context and identifying the corresponding function;
  - understand the difference between finite and infinite sequences; and
  - explore how and when to use a recursive definition for a given pattern or sequence.

- explore rates of change;
  - compare graphs of functions that have a constant rate of change (i.e., slope) versus graphs that have variable rates of change;
  - compare rates of change of linear, quadratic, square root, and other function families; and
  - explore average rates of change in regard to speed, cost, revenue, and other real-world applications.

- determine graphically and algebraically whether a nonlinear function has symmetry;
  - interpret if a given function has symmetry.

- understand that in any equation x can be interpreted as the equation $f(x) = g(x)$;
  - interpret the solutions as the x-value(s) of the intersection points of the graphs of $y_1 = f(x)$ and $y_2 = g(x)$;
  - use algebra to find the value of x that makes $f(x) = g(x)$ true; and
  - understand that functions are equal if they have the same domain and rule of correspondence.

- simplify algebraic expressions involving square roots.

- perform mathematical operations with square roots;
  - understand when to rationalize a denominator;
  - comprehend the equivalence of a simplified square root expression and the equivalence of a nonsimplified square root expression.

- add, subtract, and multiply polynomials.

- add, subtract, multiply, and divide rational algebraic expressions.

- factor expressions involving the difference/sum of two squares and trinomials in the form $ax^2 + bx + c = 0$, and factor methods limited to the greatest common factor, grouping, trial and error, and special products.

- use either factorization or square roots to solve quadratic equations in the form $ax^2 + bx + c = 0$, where $a = 1$.

- solve simple radical equations by isolating the variable and squaring both sides.

- use technology, tables, and graphs to solve equations resulting from the investigation of $x^2 + bx + c = 0$;
  - interpret the solution of a quadratic function from a graph of the data; and
  - identify and comprehend the meaning of the *x*-intercepts from a table of quadratic data.

- solve simple rational equations that result in linear or quadratic equations.

- write square roots of negative numbers in imaginary form; write complex numbers in the form $a + bi$ in the context of solving quadratic equations.

- add, subtract, multiply, and divide complex numbers;
  - apply the associative, distributive, and commutative properties; and
  - identify and find conjugates of complex numbers.

- investigate and explain characteristics of a variety of piecewise-defined functions, such as absolute value and greatest integer functions; relate these characteristics to a real-life situation modeled by such a function
  - translate fluently between graphical, algebraic, and numeric representations;
  - identify the domain and range;
  - find the vertex and axis of symmetry;
  - identify the zeroes;
  - find the *x*- and *y*-intercepts;
  - identify points of discontinuity;
  - identify intervals where the value of a function is constant, increasing, or decreasing; and
  - investigate rates of change for specified intervals.

- solve absolute value equations and inequalities;
  - use algebraic and analytical methods; and
  - determine solutions using graphs and/or number lines,

- extend properties of exponents to include all integer exponents and use expressions with integer exponents to model real-world functional relationships;
  - apply product of powers, quotient of powers, power of a power, power of a product, and power of a quotient to simplify and/or evaluate expressions; and understand that for any real number a, $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$ and apply these properties.

- investigate and explain characteristics of exponential functions; use these characteristics to model and solve real-world problems;
  - identify domain and range;
  - identify zeroes;
  - find *x*- and *y*-intercepts;
  - recognize and/or determine intervals where the value of a function is increasing or decreasing;
  - find maximum and minimum values;
  - investigate rates of change over intervals; and
  - recognize and explain behavior at extremes.

- graph exponential functions as transformations of $f(x) = a^x$;
  - recognize and use transformations of $f(x) = a^x$; and
  - use tables of value.

- solve simple exponential equations and inequalities;
  - by using algebraic and analytical methods; and
  - by reading and interpreting graphs.

- understand and recognize geometric sequences as exponential functions whose domains are the sequence of natural (counting) numbers;
  - interpret the constant ratio in a geometric sequence as the base of the associated exponential function; and
  - recognize and use concepts such as the common ratio and powers of the common ratio to solve real-world problems involving exponential growth and decay.

- convert between standard $y = (ax^2 + bx + c)$ and vertex $y = a(x - h)^2 + k$ forms of a quadratic function using the roots of the quadratic and the symmetry properties of the parabola; use the vertex form to locate and graph a quadratic function, e.g., when using a quadratic function to model a data relationship; translate from vertex form back to standard form to identify the parameters *a*, *b*, and *c*.

- graph quadratic functions as transformations of the function $f(x) = x^2$
  - identify vertical and horizontal stretches and compressions, and vertical and horizontal translations; and
  - explore reflections across the *x*- and *y*-axes.

- investigate and explain characteristics of quadratic functions; use these characteristics to model and solve real-world problems;
  - identify domain and range;
  - identify the vertex and axis of symmetry;
  - find all zeroes;
  - find the *x*- and *y*-intercepts;
  - locate extrema using ordered pairs and be able to identify maximum and minimum values;
  - determine intervals of increase and decrease; and
  - investigate rates of change for specific intervals.

- find real and complex solutions of quadratic equations analytically; be familiar with multiple methods and recognize when a certain method is most appropriate;
  - use factoring methods and the zero-product property;
  - apply the quadratic formula;
  - explore both exact and approximate solutions; recognize when each type of solution is appropriate and why; and
  - recognize how the solutions of quadratic equations apply to a real-world situation modeled by the quadratic function, e.g., when one or both roots are meaningless in context.

- understand the relationship between a function and its inverse;
  - recognize and find the inverse of a function or relation using a variety of methods: interchanging the first and second coordinates of each ordered pair; using analytical (algebraic) techniques; and/or determining that two functions, *f* and *g*, are inverses by recognizing that $(f \circ g)(x) = (g \circ f)(x) = I(x) = x$;
  - recognize that the domain of the inverse is the range of the original relation and vice versa; and
  - understand how and why domain restrictions come into play with inverse functions and relate them to the behavior of the original function.

- extend properties of exponents to include rational exponents.

- investigate and explain characteristics of exponential and logarithmic functions including domain and range, asymptotes, zeros, intercepts, intervals of increase and decrease, and rate of change.

- represent a system of linear equations as a matrix equation.

Mathematics

Domain: Geometry (approximately 36% of the test)

Overview of the Domain

- Students will investigate properties of geometric figures in a coordinate plane.
- Students will understand and use the language of mathematical argument and justification.
- Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
- Students will identify and use special right triangles.
- Students will define and apply sine, cosine, and tangent ratios to right triangles.
- Students will understand and apply the properties of circles and their associated segments and angles.
- Students will find and compare the measures of spheres.

Associated GPS

MM1G1a,c,d,e	MM1G2a,b	MM1G3a,b,c,d,e	MM2G1a,b	MM2G2c
MM2G3b,c	MM2G4a,b	MM3G1a		

Associated GPS Concepts and Skills

Assessment of this domain will focus on student ability to

- determine the distance between two points on a coordinate grid;
  - find distances between two points on the same horizontal or vertical line; and
  - use various methods (such as the distance formula or Pythagorean theorem) to calculate the distance when given two points with coordinates (x_1, y_1) and (x_2, y_2).

- calculate the midpoint of a segment;
  - use various methods (such as the midpoint formula, similar triangles, averaging the endpoints, etc.) to locate the midpoint when given two points on a coordinate grid with coordinates (x_1, y_1) and (x_2, y_2); and
  - find an endpoint of a line segment when given its other endpoint and midpoint.

- understand the distance formula as an application of the Pythagorean theorem;
  - explore how the distance formula is derived from the Pythagorean theorem; and
  - find the length of a hypotenuse or a leg of a triangle plotted on a coordinate grid.

- use the coordinate plane to investigate properties of and verify conjectures related to triangles and quadrilaterals;
  - use relationship properties of side measures, slopes, diagonals, etc., of triangles and quadrilaterals to determine unknown side lengths;
  - use side and angle theorems to prove triangles and quadrilaterals are similar and/or congruent;
  - understand the minimal information necessary to conclude that two triangles are congruent;
  - utilize properties of parallel and perpendicular lines and angle bisectors to construct or draw the missing measure of a polygon, given a known relationship to another triangle or quadrilateral;
  - utilize the distance formula to classify figures as triangles and quadrilaterals (e.g., squares, rectangles, trapezoids, kites, parallelograms, and rhombuses); and
  - determine missing vertices of a triangle or a quadrilateral by utilizing side and angle relationships of a given figure.

- use conjecture, inductive reasoning, deductive reasoning, counterexamples, and indirect proof, as appropriate, in mathematical and real-world applications;
  - utilize prior knowledge of quadrilateral relationships to prove or disprove classification of quadrilaterals; and
  - utilize paragraph proofs, flow proofs, two-column proofs, or any other method that relays clear communication to justify conclusions regarding polygon relationships.

- explore and use the relationships among conditional statements;
  - determine the hypothesis and conclusion of a conditional statement, in word or in mathematical form;
  - write the converse of a conditional statement by exchanging the hypothesis and conclusion;
  - realize that the inverse of a conditional statement is the negation of the hypothesis and conclusion of the conditional statement;
  - understand that the contrapositive of a conditional statement is the negation of the hypothesis and conclusion of the conditional statement and then the interchange of the hypothesis and conclusion; and
  - utilize conditional statements to prove algebraic, geometric, and real-world concepts.

- determine the sum of interior and exterior angles in a polygon;
  - utilize angle relationships of a polygon to find a missing measure or the total interior angles measures of a specific polygon; and
  - utilize angle relationships, such as linear pairs and the exterior angle sum theorem, to determine an exterior angle of a polygon.

- understand inequality theorems involving triangles;
  - apply the triangle inequality theorem to determine if given side lengths form a triangle;
  - utilize the side-angle inequality theorem to determine the largest and smallest angle or side in a triangle; and
  - use the exterior-angle inequality theorem, linear pairs, or the sum of the angles of a triangle adding to 180° to determine the measure of an exterior angle of a triangle when given two remote interior angles.

- understand congruence postulates and theorems for triangles;
  - identify and use SSS, SAS, ASA, AAS, HL to prove/justify that given triangles are congruent through proofs including two-column, paragraph, and flow chart, or any other valid form of communication; and
  - understand that SSA and AAA are not valid methods to prove triangle congruency.

- use and prove properties of and relationships among the following special quadrilaterals:
  - parallelograms—understand that the opposite sides are congruent, the opposite angles are congruent, the consecutive angles are supplementary, and the diagonals bisect each other;
  - rectangles—understand that the diagonals are congruent and that rectangles have all the properties of a parallelogram;
  - rhombuses—understand that the diagonals are perpendicular and bisect a pair of opposite angles and that rhombuses have all the properties of a parallelogram;
  - squares—understand that the diagonals are perpendicular and congruent and that squares have all the properties of a parallelogram;
  - isosceles trapezoids—understand that they have only one pair of parallel sides and congruent diagonals; and
  - kites—understand that the diagonals are perpendicular and that one diagonal is bisected, or the opposite sides are congruent and have congruent and perpendicular diagonals, and that kites have all the properties of a parallelogram.

- find and use points of concurrency, such as incenter, orthocenter, circumcenter, and centroid, in triangles;
  - use bisectors, medians, and altitudes to find points of concurrency;
  - locate centers of circles inscribed in or circumscribed about triangles; and
  - make decisions about which center best meets a given set of conditions.

- determine the lengths of sides of 30°-60°-90° triangles;
  - use the fact that the length of the hypotenuse is twice the length of the shorter leg and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg to determine the lengths of all three sides given any one of the three sides; and
  - solve problems that involve application of these side length relationships.

- determine the lengths of sides of 45°-45°-90° triangles;
  - use the fact that the length of the hypotenuse is $\sqrt{2}$ times the length of each leg to determine the lengths of all sides of a triangle given the length of any one of the three sides; and
  - solve problems that involve application of these side length relationships.

- understand and apply the basic trigonometric ratios for right triangles.

- understand and use properties of and relationships among angles related to circles, such as central, inscribed, and related angles, e.g.,
  - relationship between arc measures and angle measures;
  - relationship between measures of central angles and inscribed angles; and
  - relationship of angles created by a chord with a common endpoint on the circle and the line that is tangent at that point; a secant and a tangent; two secants; and two tangents.

- use the properties of circles to solve problems involving the length of an arc and the area of a sector.

- understand, use and apply the surface area and volume of a sphere;
  - calculate surface area and volume of a sphere;
  - find the radius, diameter, and/or circumference of a sphere given the volume or a relationship from which the volume can be determined; and
  - determine the effect on surface area and volume when changing the radius or diameter of a sphere or vice versa.

- Find equations of circles.

Mathematics

Domain: Data Analysis (approximately 28% of the test)

Overview of the Domain

- Students will determine the number of outcomes related to a given event.
- Students will use the basic laws of probability.
- Students will relate samples to a population.
- Students will explore variability of data by determining the mean absolute deviation (the average of the absolute values of the deviations).
- Students will use sample data to make informal inferences using population means and standard deviations.
- Students will determine an algebraic (limited to linear or quadratic) model to quantify the association between two quantitative variables.

Associated GPS

MM1D1a,b MM1D2a,b,c,d MM1D3a,c MM2D1a,c MM2D2b,d

Associated GPS Concepts and Skills

Assessment of this domain will focus on student ability to

- apply the addition and multiplication principles of counting.

- calculate and use simple permutations and combinations;
  - integrate the multiplication principle to clarify the difference between permutations and combinations and when each is appropriate to use for a situation;
  - use diagrams to justify the classification; and
  - utilize permutation and combination formulas to determine the number of possible arrangements of real-world events.

- understand when an event is mutually exclusive and use diagrams, tables, and the formula $P(A \text{ or } B) = P(A) + P(B)$ to calculate the probability of mutually exclusive events.
- use diagrams, tables, and the formula $P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$ to find the probabilities of dependent events and understand when an event is dependent.
- use diagrams, tables, and the formula $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$ to calculate conditional probabilities of real-world events.

- use expected value to predict outcomes and make inferences.

- compare summary statistics from one sample data distribution to another sample data distribution;
  - interpret the mean, median, quartiles, and interquartile range of multiple data sets;
  - understand normal and binomial data distributions; and
  - describe center and variability of data distributions.

- understand that a random sample is used to improve the chance of selecting a representative sample;
  - determine the type of sampling to be used, given a scenario, so that a survey yields results from a random population sample; and
  - understand that a random sample will yield unbiased results.

- recognize an appropriate question given a research topic and populations of interest;
  - identify potential bias created by questions.

- use means and standard deviations to compare data sets;
  - understand and apply various strategies for estimating means and standard deviations for comparison purposes;
  - understand various representations of data, including tables, graphs, line plots, stem-and-leaf plots, histograms, and box-and-whisker plots; know which information can be directly determined and which can only be estimated from a given representation; and
  - understand the role of n in comparing standard deviations of data sets, including recognizing when n is unknown.

- examine the issues of curve fitting by finding good linear fits to data using simple methods such as the median-median line by "eyeballing;"
  - decide whether a linear or quadratic model or neither is appropriate for data presented in a table or graph;
  - recognize an appropriate algebraic model given a table or graph (note: in the absence of appropriate technology, students will be expected to estimate the correct parameters for a linear or quadratic function presented in the form $y = ax + b$ or $y = ax^2 + bx + c$); and
  - decide whether or not a particular set of data is appropriately modeled by a given function.

- understand issues that arise when using data to explore the relationship between two variables, including correlation, e.g., recognizing whether the fit of an algebraic model is strong, weak, or nonexistent; focus on confusion between correlation and causation.