Mathematics Common Core Georgia Performance Standards

K-12 Mathematics Introduction

The Georgia Mathematics Curriculum focuses on actively engaging the students in the development of mathematical understanding by using manipulatives and a variety of representations, working independently and cooperatively to solve problems, estimating and computing efficiently, and conducting investigations and recording findings. There is a shift towards applying mathematical concepts and skills in the context of authentic problems and for the student to understand concepts rather than merely follow a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different ways to a solution and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things well, via reasoning, permit students to know much else—without having to commit the information to memory as a separate fact. It is the connections, the reasoned, logical connections that make mathematics manageable. As a result, implementation of the Common Core Georgia Performance Standards places a greater emphasis on problem solving, reasoning, representation, connections, and communication.

Georgia Performance Standards
History of Mathematics

This is a one-semester elective course option for students who have completed AP Calculus or are taking AP Calculus concurrently. It traces the development of major branches of mathematics throughout history, specifically algebra, geometry, number theory, and methods of proofs, how that development was influenced by the needs of various cultures, and how the mathematics in turn influenced culture. The course extends the numbers and counting, algebra, geometry, and data analysis and probability strands from previous courses, and includes a new history strand.

(*Prerequisite: Successful completion of Accelerated Mathematics III or Mathematics IV*)

Instruction and assessment should include appropriate use of technology and manipulatives. Concepts should be introduced and used in an appropriate historical context.

<u>NUMBERS AND OPERATIONS</u>

Students will investigate historical computation algorithms and use them to solve problems; define and explore the concepts of denumerability and algebraic numbers.

MHMN1. Students will explore and use historical computational methods.

a. Use Babylonian, Roman, Egyptian (hieratic and hieroglyphic), Chinese, and Greek number systems to represent quantities.

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b. Use historical multiplication and division algorithms (including the Egyptian method of duplation and mediation, the medieval method of gelosia, and Napier's rods).

MHMN2. Students will explore the implications of infinite sets of real numbers.

a. Describe denumerable and nondenumerable sets and provide examples of each.
b. Identify algebraic and transcendental numbers.

<u>ALGEBRA</u>

Students will investigate historical equation solving techniques in both algebraic and geometric form; understand and use the axiomatic method of abstract algebra; compute defined products on sets of complex numbers; solve linear congruences; determine whether a quadratic congruence has a solution; understand and use basic number-theoretic concepts.

MHMA1. Students will explore and use historical methods for expressing and solving equations.

a. Solve linear equations using the method of false position.
b. Express the geometrical algebra found in historical works (such as the Elements of Euclid) in modern algebraic notation.
c. Solve systems of linear and nonlinear equations using Diophantus' method.
d. Translate into modern notation problems appearing in ancient and medieval texts that involve linear, quadratic, or cubic equations and solve them.
e. Use Cardano's cubic formula and Khayyam's gemoetric construction to find a solution to a cubic equation.

MHMA2. Students will explore abstract algebra and group-theoretic concepts.

a. Add, subtract, and multiply two quaternions.
b. Explore matrix products other than the Cayley product (including Lie and Jordan) by determining whether these products are associative or commutative.
c. Identify whether a given set with a binary operation is a group.

MHMA3. Students will use and apply number theoretic concepts.

a. Find the first four perfect numbers using Euclid's formula.
b. Prove statements concerning figurate numbers using both graphical (as in the manner of the Greeks) and algebraic methods.
c. Solve simple linear congruences of the form $ax = b \bmod m$.
d. Use Fermat's Little Theorem and Euler's Theorem to simplify expressions of the form $a^k \bmod m$.
e. Use Gauss' Law of Quadratic Reciprocity to determine quadratic residues of two odd primes; i.e., solve quadratic congruences of the form $x^2 = p \bmod q$.
f. Discover that the real primes that can be expressed as the sum of two squares are no longer prime in the field of Gaussian integers.

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MHMA4. Students will use the algebraic techniques of Fermat, Barrow, and Newton to determine tangents to quadratic curves.

GEOMETRY

Students will prove basic Euclidean propositions and constructions; compute lengths, areas, and volumes according to historical algorithms and formulae; understand and prove basic non-Euclidean propositions.

MHMG1. Students will prove geometry theorems.

a. Students will understand and recognize the use of definitions, postulates, and axioms in defining a deductive system such as Euclidean geometry.
b. Prove the first five propositions in Book I of Euclid's *Elements*.
c. Construct a regular pentagon with a straight-edge and compass.

MHMG2. Students will compute lengths, areas, and volumes according to historical formulas.

a. Find the volume of a truncated pyramid using the Babylonian, Chinese, and Egyptian formulas.
b. Compute the areas of regular polygons by Heron's formulas.
c. Identify cyclic quadrilaterals and find associated lengths by Ptolemy's Theorem.

MHMG3. Students will explore and prove statements in non-Euclidean geometry.

a. Prove that the summit angles of an isosceles birectangle are congruent, but that it is impossible to prove they are right without referring to the parallel postulate or one of its consequences.
b. Describe the hypothesis of the acute angle (Hyperbolic), the hypothesis of the right angle (Euclidean), and the hypothesis of the obtuse angle (Spherical).
c. Prove that under the hypothesis of the acute angle, similarity implies congruence.

DATA ANALYSIS AND PROBABILITY

Students will explore the origins of probability by solving problems concerning gambling.

MHMD1. Students will compute the ratio of winnings in an interrupted game.

HISTORY

Students will understand the influence Hindu-Arabic numerals had on the development of mathematics; recognize the accomplishments of the ancient Greeks and the influence ancient Greek mathematical ideas had on their culture and later cultures; understand how mathematical ideas were preserved and extended after Greek society collapsed; investigate the development of algebra during the Middle Ages and Renaissance; understand how both algebraic and geometrical ideas influenced the development of analysis; identify the influences analysis had on our understanding of science; recognize the move toward abstraction in the 19th and 20th centuries is an extension of the ancient Greek conception of proof.

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MHMH1. Students will identify Hindu-Arabic numerals as a prime scientific advancement.

a. Describe the limitations of the Babylonian, Roman, Egyptian (hieratic and hieroglyphic), Chinese, and Greek number systems as compared to Hindu-Arabic numerals.
b. Describe the transition of Hindu-Arabic numerals from regional use in the 10th century to wide-spread use in the 15th (including the influence of Fibonacci for the use of the numerals and the Italian abascists against their use).
c. Identify the number system and notation used by a society as an influence on the types of mathematics developed by that society.

MHMH2. Students will describe factors involved in the rise and fall of ancient Greek society.

a. Describe the theories for the rise of intellectual thought in ancient Greece and the factors involved in its collapse.
b. Describe the cultural aspects of Greek society that influenced the way mathematics developed in ancient Greece.
c. Explain the distinction made between number and magnitude, commensurable and incommensurable, and arithmetic and logistic, the cultural factors inherent in this distinction, and the logical crisis that occurred concerning incommensurable (irrational) magnitudes.

MHMH3. Students will trace the centers of development of mathematical ideas from the 5th century to the 18th century.

a. Describe the transmission of ideas from the Greeks, through the Islamic peoples, to medieval Europe.
b. Describe the influence of the Catholic Church and Charlemagne on the establishment of mathematics as one of the central pillars of education.
c. Explain the cultural factors that encouraged the development of algebra in 15th century Italy, and how this development influenced mathematical thought throughout Europe.
d. Identify the works of Galileo, Copernicus, and Kepler as a landmark in scientific thought, describe the conflict between their explanation of the workings of the solar system and then-current perspectives, and contrast their works to those of Aristotle.
e. Describe the contributions of Fermat, Pascal, Descartes, Newton, and Gauss to mathematics.
f. Identify Euler as the first modern mathematician and a motivating force behind all aspects of mathematics for the 18th century.
g. Describe the influence the French Revolution had on education (establishment of the Ecole Normale and the Ecole Polytechnique, Monge, Lagrange, Legendre, Laplace).

MHMH4. Students will identify the 19th and 20th centuries as the time when mathematics became more specialized and more rigorous.

a. Describe the societal factors that inhibited the developement of non-Euclidean geometry.
b. Explain how the ancient Greek pattern of material axiomatics evolved into abstract axiomatics (non-Euclidean geometry, non-commutative algebra)
c. Identify Cantor as the most original mathematician since the ancient Greeks.

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d. Describe the implications of Klein's *Erlanger Programme* and Godel's Incompleteness Theorem on the nature of mathematical discovery and proof.

Terms/Symbols: duplation and mediation, gelosia, Napier's rods, unit fraction, denumerable set, nondenumerable set, algebraic number, transcendental number, false position, quaternion, group, perfect number, figurate number, linear congruence, quadratic congruence, quadratic residue, Gaussian integer, cyclic quadrilateral, non-Euclidean geometry

Process Standards

The following process standards are essential to mastering each of the mathematics content standards. They emphasize critical dimensions of the mathematical proficiency that all students need.

MHMP1. Students will solve problems (using appropriate technology).

a. Build new mathematical knowledge through problem solving.
b. Solve problems that arise in mathematics and in other contexts.
c. Apply and adapt a variety of appropriate strategies to solve problems.
d. Monitor and reflect on the process of mathematical problem solving.

MHMP2. Students will reason and evaluate mathematical arguments.

a. Recognize reasoning and proof as fundamental aspects of mathematics.
b. Make and investigate mathematical conjectures.
c. Develop and evaluate mathematical arguments and proofs.
d. Select and use various types of reasoning and methods of proof.

MHMP3. Students will communicate mathematically.

a. Organize and consolidate their mathematical thinking through communication.
b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
c. Analyze and evaluate the mathematical thinking and strategies of others.
d. Use the language of mathematics to express mathematical ideas precisely.

MHMP4. Students will make connections among mathematical ideas and to other disciplines.

a. Recognize and use connections among mathematical ideas.
b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
c. Recognize and apply mathematics in contexts outside of mathematics.

MHMP5. Students will represent mathematics in multiple ways.

a. Create and use representations to organize, record, and communicate mathematical ideas.
b. Select, apply, and translate among mathematical representations to solve problems.
c. Use representations to model and interpret physical, social, and mathematical phenomena.

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Reading Standard Comment
After the elementary years, students are seriously engaged in reading for learning. This process sweeps across all disciplinary domains, extending even to the area of personal learning. Students encounter a variety of informational as well as fictional texts, and they experience text in all genres and modes of discourse. In the study of various disciplines of learning (language arts, mathematics, science, social studies), students must learn through reading the communities of discourse of each of those disciplines. Each subject has its own specific vocabulary, and for students to excel in all subjects, they must learn the specific vocabulary of those subject areas *in context.*
Beginning with the middle grades years, students begin to self-select reading materials based on personal interests established through classroom learning. Students become curious about science, mathematics, history, and literature as they form contexts for those subjects related to their personal and classroom experiences. As students explore academic areas through reading, they develop favorite subjects and become confident in their verbal discourse about those subjects. Reading across curriculum content develops both academic and personal interests in students. As students read, they develop both content and contextual vocabulary. They also build good habits for reading, researching, and learning. The Reading Across the Curriculum standard focuses on the academic and personal skills students acquire as they read in all areas of learning.

MRC. Students will enhance reading in all curriculum areas by:

a. Reading in all curriculum areas
- Read a minimum of 25 grade-level appropriate books per year from a variety of subject disciplines and participate in discussions related to curricular learning in all areas
- Read both informational and fictional texts in a variety of genres and modes of discourse
- Read technical texts related to various subject areas

b. Discussing books
- Discuss messages and themes from books in all subject areas.
- Respond to a variety of texts in multiple modes of discourse.
- Relate messages and themes from one subject area to messages and themes in another area.
- Evaluate the merit of texts in every subject discipline.
- Examine author's purpose in writing.
- Recognize the features of disciplinary texts.

c. Building vocabulary knowledge
- Demonstrate an understanding of contextual vocabulary in various subjects.
- Use content vocabulary in writing and speaking.
- Explore understanding of new words found in subject area texts.

d. Establishing context
- Explore life experiences related to subject area content.
- Discuss in both writing and speaking how certain words are subject area related.
- Determine strategies for finding content and contextual meaning for unknown words.

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