Student Name: _____

Georgia High School Graduation Tests

GPS Version

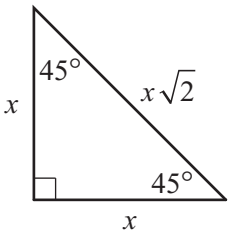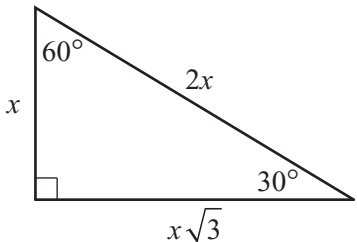# Mathematics

Sample Items
with Commentary

GEORGIA
DEPARTMENT OF
EDUCATION

Georgia High School Graduation Tests
Mathematics Formula Sheet

Below are the formulas you may find useful as you work the problems. However, some of the formulas may not be used. You may refer to this page as you take the test.

Area

Rectangle/Parallelogram $A = bh$

Triangle $A = \dfrac{1}{2}bh$

Circle $A = \pi r^2$

Trapezoid $A = \dfrac{1}{2}(h)(b_1 + b_2)$

Circumference

$C = \pi d \quad \pi \approx 3.14$

Volume

Rectangular Prism/Cylinder $V = Bh$

Pyramid/Cone $V = \dfrac{1}{3}Bh$

Sphere $V = \dfrac{4}{3}\pi r^3$

Surface Area

Rectangular Prism $SA = 2lw + 2wh + 2lh$

Cylinder $SA = 2\pi r^2 + 2\pi rh$

Sphere $SA = 4\pi r^2$

Trigonometric Relationships

$\sin(\theta) = \dfrac{\text{opp}}{\text{hyp}}; \quad \cos(\theta) = \dfrac{\text{adj}}{\text{hyp}}; \quad \tan(\theta) = \dfrac{\text{opp}}{\text{adj}}$

Pythagorean Theorem

$a^2 + b^2 = c^2$

Quadratic Formula

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Standard Form $ax^2 + bx + c = y$

Vertex Form $a(x - h)^2 + k = y$

Expected Value

$E(x) = \displaystyle\sum_{i=1}^{n} x_i p(x_i)$

the sum of each outcome multiplied by its probability of occurrence

Permutations

$_nP_r = \dfrac{n!}{(n - r)!}$

Combinations

$_nC_r = \dfrac{n!}{r!(n - r)!}$

Interquartile Range

the difference between the first quartile and third quartile of a set of data

Special Right Triangles

45°– 45°– 90° Triangle

30°– 60°– 90° Triangle

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
1	2A,1,b; P,1,b; P,4,b	2	D

1. The function $g(x) = |x - 5|$ is the result of a translation of the function $f(x) = |x|$. How is the graph of $g(x)$ different from the graph of $f(x)$?

    A. The graph of $g(x)$ is 5 units up.

    B. The graph of $g(x)$ is 5 units down.

    C. The graph of $g(x)$ is 5 units to the left.

    D. The graph of $g(x)$ is 5 units to the right.

COMMENTARY:
In this item students are asked to recognize a horizontal translation from the algebraic statement of the function and to interpret the effect this translation will have on the graph of the new function. Students who have had extensive use of the graphing calculator during classroom instruction and experience linking the graph to an algebraic form in classroom activities and exercises will have little trouble with this question. Students who have not had the multiple examples and experiences that the use of a graphing calculator provides may struggle with this simple question. However it is not necessary to use a graphing calculator to answer this question. Answers A and B are incorrect because the given translation is not vertical. Answer C is incorrect because the translation is not to the left. Answer D is correct.

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
2	1A,2,a; P,1,b	1	D

2. Which expression is equivalent to $\sqrt{32b^{16}}$?

    A. $16b^4$

    B. $16b^8$

    C. $4b^4\sqrt{2}$

   D. $4b^8\sqrt{2}$

COMMENTARY:
This item asks students to simplify an expression containing a radical. GPS Mathematics emphasizes a balance of concepts, skills, and problem solving; this question exemplifies the skills part of the curriculum. Answer A is incorrect because students used the exponent algorithm on the coefficient and square root on the exponent, the exact reverse of what they should have done. Answer B is incorrect because the coefficient was treated as an exponent. Answer C is incorrect because the square root of the exponent was taken. Answer D is correct.

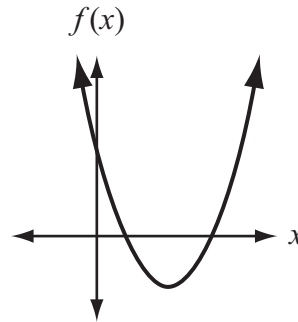| ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
3	1A,3,c; P,5,c	2	A

3. A student is studying the quadratic function f. The student determined that $f(0) > 0$. The student also determined that f has two real roots, a and b, such that $a < b < 0$.
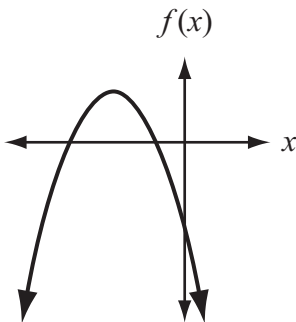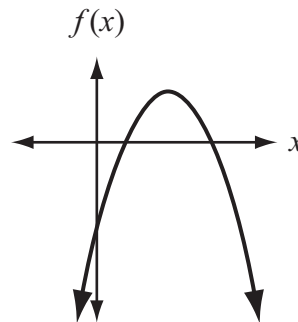
Which graph could represent f?

A. $f(x)$

B. $f(x)$

C. $f(x)$

D. $f(x)$
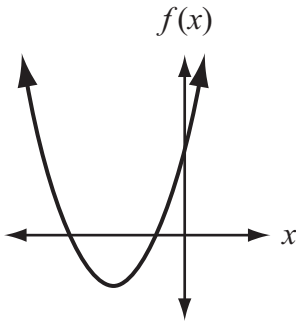
COMMENTARY:
In this item students are asked to interpret given characteristics to identify the matching graph. The first piece of information is that f is a quadratic function. Students who know what the graph of the parent function looks like are well prepared for recognizing options that differ in their basic graph form. However, given the choices in this item, this information does not help lead students to the correct response. Students experienced with making this first conclusion will be better prepared for choices that differ in their basic graph form. The second piece of information is the $f(0) > 0$. Many students cannot put this inequality statement into a meaningful mathematics description. Students should recognize that $f(0)$ refers to the y-intercept and that > 0 means positive value, so the inequality statement tells us that the y-intercept is positive. Reading the statement $f(0) > 0$ and making the connection to the statement the y-intercept is positive is an important translation for students. This piece of information eliminates choices C and D. The last piece of given information tells us that f has two real roots and that $a < b < 0$. Students need to be able to interpret the inequality statement in order to identify the graph easily. The two roots are both negative and distinct (since one is smaller than the other). That piece of information eliminates choice B since both of these roots are positive leaving choice A, which does fit all the given conditions. Frequent in-class investigations will prepare students for this type of analysis which requires verbalization and examination of thought processes.

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
4	2A,1,c; P,1,b	1	A

4. A student drew this graph of the function f.
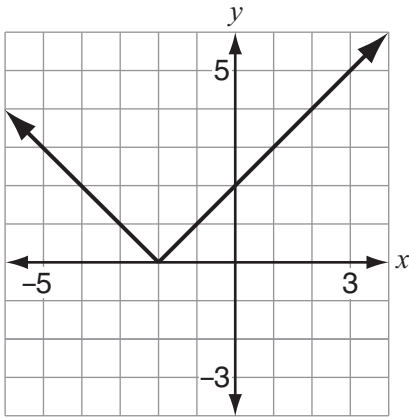

Which value of x satisfies $f(x) = 1$?

A. $x = -3$

B. $x = -2$

C. $x = 1$

D. $x = 3$

COMMENTARY:
This item gives students the graph of a function and asks them to identify a value of the domain that meets the given condition. Students must know how to interpret the symbolic information given. They must recognize that $f(x) = 1$ is asking for the x value of the ordered pair on the graph when y has a value of 1. The students must next identify the two places on the graph where the y-value is 1 and finally, by looking at the choices given, choose the correct answer. Answer A is correct because the point $(-3, 1)$ does lie on the graph of the given function. Answer B is incorrect because it is the x-intercept and the vertex of the graph, but the y-value at -2 is 0 not 1. Answer C is incorrect because it assigns the value of 1 to the x-coordinate, not the y-coordinate. Answer D is incorrect because it confuses the x- and y- coordinates of the points and gives the y-value (3) when the x-value is 1. Students who cannot differentiate coordinates are likely to choose D.

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
5	2A,4,b; P,1,b	2	A

5. The Georgia state flag consists of a square
    and three rectangles. Each rectangle has the
    same width, x. The length of each of the two
    smaller rectangles is equal to $3x$, as shown
    in this diagram.
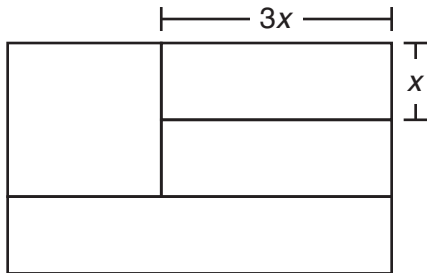
The area of this particular Georgia flag is
60 square feet. What is the length of x?

A. 2 feet

B. 4 feet

C. $2\sqrt{5}$ feet

D. $2\sqrt{15}$ feet

COMMENTARY:
This item illustrates the seamless integration of algebra and geometry and why making those
connections is important all the way through mathematics. Students must use their knowledge of
properties of squares and rectangles to make appropriate conclusions about the dimensions of the parts
of and the entire Georgia state flag. The measurements of two pieces are given, because each rectangle
has the same width and two stripes or widths make up one side of the square, students should conclude
that the side of the square is $2x$. This makes the dimensions of the entire flag $5x$ by $3x$. Once these
dimensions are known, students multiply and set the product equal to the area of the flag and solve for
x using square roots. Answer A is correct. Answer B is incorrect because the student did not have the
square of x in the equation or solved the x squared instead of x. Answer C is incorrect because the
student found the area of the labeled rectangle $(3x)(x)$ set that equal to 60 and solved correctly. Answer
D is incorrect because the student simply found the square root of 60.

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
6	1G,2,b; P,3,d	2	A

6. Which of these true statements also has a
   true inverse?

   A. If the product of integers a and b is
      odd, then both a and b are odd.

   B. If x is a multiple of 6, then x is an
      even number.

   C. If a and b are consecutive integers,
      then the sum of a and b is odd.

   D. If p is negative, then $\left|-p\right|$ is positive.

COMMENTARY:
In this item, students are asked to write and evaluate the truthfulness of the inverse of a conditional statement. This problem situation emphasizes the importance of having students routinely verbally express, analyze, and defend their thinking processes. Students who have had experiences talking (and thus demonstrating the thinking process) through these types of situations will find this question quite easy. Answer A's inverse would be: If the product of integers a and b is not odd, then both a and b are not odd; however, the product of 2 and 3 is not odd, and both of them are not odd, and further the product of one even and one odd is always even, so answer A is correct. Answer B's inverse would be: If x is not a multiple of 6, then x is not an even number; however, 8 is not a multiple of 6 but it is even, so answer B is incorrect. Answer C's inverse would be: If a and b are not consecutive integers, then the sum of a and b is not odd; however, 6 and 9 are not consecutive integers, but their sum is odd, so answer C is incorrect. Answer D's inverse would be: If p is not negative, then $\left|-p\right|$ is not positive; however 6 is not negative but the absolute value of negative 6 is positive, so answer D is incorrect. Encouraging students to eliminate responses B-D after selecting response is an effective test taking strategy.

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
7	1G,3,a; P,1,b; P,1,c; P,4,a	2	C

7. One interior angle of a pentagon has a measure of 120°. The other four interior angles are congruent to each other.

What is the measure of one of the four congruent angles?

A.   30°

B.   60°

C.   105°

D.   195°

COMMENTARY:
In this item, the students must remember how to find the sum of the interior angles of a polygon. For a pentagon that sum is 3(180) or 540. One angle of 120 already exists; the sum of the other four is then 420. Since the other four are congruent, the student should simply divide 420 by 4 to get the measure of each angle. Answer A is incorrect; the student divided 120 by 4. Answer B is incorrect; the students mistakenly thought that the sum of the interior angles is 360, subtracted the 120 to get 240, and then divided by 4. Answer C is correct. Answer D is incorrect; the student thought the sum of the interior angles was the product of the number of sides times 180, which equaled 900. The student then subtracted the 120 and divided by 4.

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
8	2G,1,b; P,1,b; P,1,c; P,4,a	2	B

8. This diagram shows a square tile with a
diagonal length of 16 inches.


16 in.
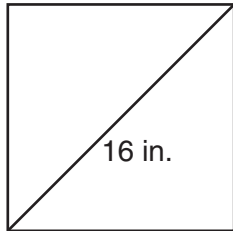
What is the approximate area of the tile?

A. 64 square inches

B. 128 square inches

C. 181 square inches

D. 256 square inches

COMMENTARY:
This item requires a student to recognize and use the properties of the sides, angles, and diagonals of a square. There are multiple ways a student could have worked this problem. The most obvious is to recognize the triangle shown as an isosceles right triangle, use the hypotenuse given to find the leg of the triangle, which is also the side of the square, and then square the length of the side to find the area. A less obvious way is to recognize that a square is a rhombus with congruent diagonals and use the rhombus area formula which is one-half the product of the diagonals (or since the diagonals of a square are congruent, one-half the square of the diagonal). Students may even draw in the other diagonal, form four triangles and use the areas of the triangles to find the area of the square. Answer A is incorrect because the student took one-half of the diagonal and then squared. Answer B is correct. Answer C is incorrect because the student multiplied 16 times 8 times the square of 2 or they multiplied the diagonal times the length of the side. Answer D is incorrect because the student squared the diagonal but didn't divide by two.

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
9	2G,2,c; P,1,b; P,4,a	1	D

9. A student drew this diagram of a right triangle.
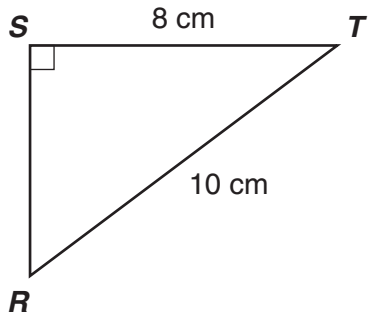

What is the value of the tangent of ∠R?

A. $\dfrac{4}{5}$

B. $\dfrac{5}{4}$

C. $\dfrac{3}{4}$

D. $\dfrac{4}{3}$

COMMENTARY:
This item requires students to perform two operations. They must know the trigonometric ratios and recognize that they need the missing side to answer the question asked. To find the missing side, students must use the Pythagorean Theorem. Answer A is incorrect because the student gave sin R not tangent. Answer B is incorrect because the student gave the reciprocal of the sin. Answer C is incorrect because the student gave tangent T not R. Answer D is correct.

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
10	2G,3,c; P,1,b	2	B

10. This circle has a radius of 9 inches.
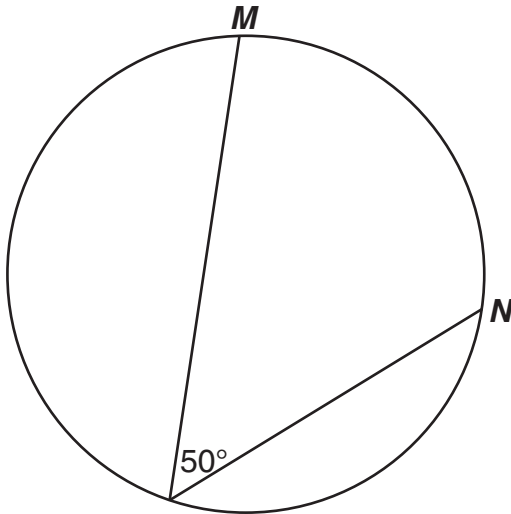
What is the approximate length of arc *MN*?

A.   8 in.

B.   16 in.

C.   23 in.

D.   35 in.

COMMENTARY:
In this item, the student must first use the relationship between the inscribed angle and its intercepted arc to find the measure of arc *MN*. Then the student will find the fractional portion of the circumference that arc *MN* represents. Answer A is incorrect because the student used the measure of the arc as equal to the measure of the inscribed angle. Answer B is correct. Answer C is incorrect because the student used three times the inscribed angle for the arc. Answer D is incorrect because the student either used four times the inscribed angle for the arc or the diameter instead of the radius.

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
11	1D,1,b; P,1,b; P,4,a; P,1,c	2	B

11. There are 10 students who applied for internships. Only 3 positions are available.

   How many different groups of 3 can be selected from the 10 students?

    A.   30
    B.   120
    C.   720
    D. 1000

COMMENTARY:
This item requires students to recognize that this problem is a combination situation and then find the number of combinations for the given numbers. Answer A is incorrect because the student used the multiplication principle and multiplied 3 and 10. Answer B in correct. Answer C is incorrect because the student thought this was a permutation situation. Answer D is incorrect because the student thought that they needed to multiply 10 times 10 times 10 or to cube 10.

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
12	1D,2,d; P,1,b; P,1,c; P,5,c	2	C

12. Jerry will spin the arrow on this spinner once.
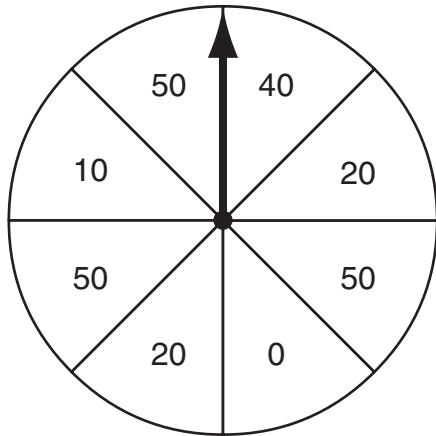
What is the expected value of Jerry's spin?

A.   20

B.   25

C.   30

D.   50

COMMENTARY:

This item requires students to understand the probability of an outcome and how to find the expected value of all the probabilities. The student must list all the outcomes that could occur from this experiment, and the probability of each outcome, then multiply each outcome by its probability and sum the results. Answer A is incorrect; it is the median of the possible outcomes without considering repeats. Answer B is incorrect; it is the middle from between 0 and 50. Answer C is correct. Answer D is incorrect; is it the mode of the outcomes.

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
13	1D,3,a; P,4,c	2	D

13. A group of 100 people were asked to rate two restaurants on a scale from 0 to 10. The results are represented by this double box-and-whisker plot.

Restaurant Ratings

Which statement is correct?

A. The range of ratings is greater for Restaurant A than for Restaurant B.

B. The range of ratings is greater for Restaurant B than for Restaurant A.

C. The interquartile range of ratings is greater for Restaurant A than for Restaurant B.

D. The interquartile range of ratings is greater for Restaurant B than for Restaurant A.

COMMENTARY:
In this item, students must understand how to read from a graph the characteristics of a box and whisker plot, specifically the range and interquartile range. Then they must compare these characteristics to answer the given question. Answers A and B are incorrect because the range is the same for both restaurants. Answer C is incorrect because the *IQR* for A is 2 and the *IQR* for B is 4, so A is not greater. Answer D is correct.

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
14	2D,1,c	2	B

14. A marketing researcher asked a random selection of adults to rate two different brands of toothpaste on a scale from 1 through 10.

- Brand X had a mean rating of 7.5 with a standard deviation of 1.1.

- Brand Y had a mean rating of 6.8 with a standard deviation of 2.0.

Based on the data, which statement **must** be true?

A. The data is more dispersed for Brand X.

B. The data is more dispersed for Brand Y.

C. The range of the data is greater for Brand X.

D. The range of the data is greater for Brand Y.

COMMENTARY:

This item requires students to understand the mean and standard deviation of two sets of data and how they compare to each other. Answer A is incorrect because the standard deviation is smaller which makes the spread, or how the data are dispersed, smaller. Answer B is correct. Answers C and D are incorrect because the ranges of the data MAY be greater based on the given information, but the statements in options C and D do not have to be true.

ITEM NUMBER	STANDARD ALIGNMENT	DOK	KEY
15	2D,2,b; P,1,b	2	B

15. A student drew this scatter plot.
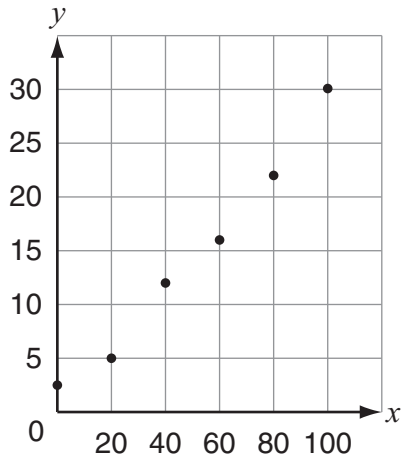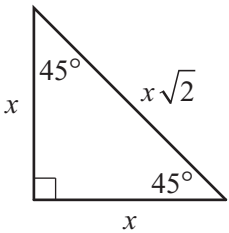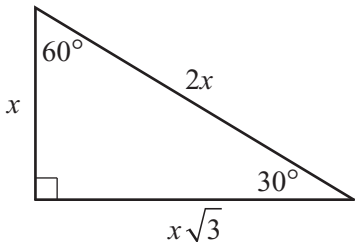

Which equation best models the data in the scatter plot?

A. $y = 0.1x + 3$

B. $y = 0.3x + 1$

C. $y = x + 0.3$

D. $y = 3x + 0.1$

COMMENTARY:
In this item, students must choose the equation of a line that best models the data given in the scatter plot. Since the data are given in a graph, students can approach this question several ways. They can use the data plot to approximate slope (noting the different scales for x and y) and since the slope of each answer choice given is different, they could identify the best choice using the slope. The students could also use the plotted ordered pairs to substitute into the given equations to determine which models the data best. Answer A is incorrect because the slope is too small and/or it does not work for all data points; though it does seem to work for the first two sets, students need to know to check the extreme data points when using substitution. Answer B is correct. Answer C is incorrect because the slope is too large; the different scales on the x and y axis may have caused this mistake. Answer D is incorrect because the slope is too large. The different scales on the x and y axis may have caused this mistake.

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