NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

NCTM

Instruction

Research Brief

Effective Strategies for Teaching Students with Difficulties in Mathematics

This research brief focuses on evidence-based practices for teaching students with difficulties in mathematics. Most of the summary for this research brief is based on two recently conducted meta-analyses (Baker, Gersten, and Lee 2002; Gersten et al. 2006) as well as complementary work by Kroesbergen and van Luitt (2003). Together, the reviews encompass more than fifty studies, and although this is an emerging and substantial research base, it is far from definitive. As a composite, the studies reviewed present a picture of specific aspects of instruction that are consistently effective in teaching students who experience difficulties with mathematics. The principles that emerged from the research seem appropriate for instruction in a variety of situations and possible settings.

Six aspects of instruction have been studied in depth. Table 1 lists each of these along with the average effect size for teaching special education students (Gersten et al. 2006) and other students with difficulties learning mathematics (Baker, Gersten, and Lee 2002). Effect sizes of 0.2 are considered small, 0.4 moderate, and 0.6 or above large. A small effect might raise students' scores on a standardized test about 8 percentile points; a large effect would raise a score approximately 25 percentile points.

Table 1

Effect Sizes for Instructional Variables for Special Education Students and Other Low-Achieving Students

Instructional Strategy	Effect Size for Special Education Students	Effect Size for Low- Achieving Students
1. Visual and graphic depictions of problems	**0.50** Moderate	NA
2. Systematic and explicit instruction	**1.19** Large	**0.58** Moderate to Large
3. Student think- alouds	**0.98** Large	NA
4. Use of structured peer-assisted learning activities involving heterogeneous ability groupings	**0.42** Moderate	**0.62** Large
5. Formative assessment data provided to teachers	**0.32** Small to Moderate	**0.51** Moderate
6. Formative assessment data provided directly to students	**0.33** Small to Moderate	**0.57** Moderate to Large

Visual and Graphic Depictions of Problems

Graphic representations of mathematical concepts and problems appeared in most commonly used textbooks. They are crucial components of programs used in nations that perform well on international comparisons, such as Singapore, Korea, or the Netherlands. Results indicate that these approaches were *moderately effective* for special education students. The average *effect* size was 0.50; the effect sizes for individual studies ranged from 0.32 to 0.88. The reviewed set of studies explored several different approaches.

An interesting finding for the use of graphics and visual organizers was that the *specificity of the visual representation determined the effectiveness of the intervention.* When teachers presented graphic depictions of problem-solving sets with multiple examples and had students practice using their own graphic organizers with specific guidance by the teacher on which visuals to select and why, the effects were much larger than when students did not have this practice or guidance.

Two recent studies of middle and high school students learning algebra and fractions add another dimension to the use of visuals with this population. The researchers in these studies developed an approach they call *concrete-representation-abstract* to teach successfully concepts and operations involving fractions (Butler et al. 2003) and basic algebra (Witzel, Mercer, and Miller 2003). Note that manipulatives are not used to skirt the teaching of the abstraction necessary to understand mathematics. Rather, they are used for a day or two so that students really understand the visual organizers and representations. The benefit of this approach may be that its intensity and concreteness help students maintain a framework in their working memory for solving problems of this type. One important finding in this body of research is the need for teachers or tutors to include some work with manipulatives for this group even in middle school and high school.

Systematic and Explicit Instruction

Consistently strong effects were found for systematic, explicit instruction. We define explicit instruction as instruction that involves a teacher demonstrating a specific plan (strategy) for solving the problem types and students using this plan to think their way through a solution.

In most studies, the emphasis was placed on providing highly explicit models of steps and procedures or questions to ask in solving problems. The degree of structure and specificity is atypical in conventional mathematics texts. We divided the explicit instruction studies into two categories: those involving only one problem type, and those involving multiple problem types. In both instances, mean effect sizes were large for both the special education students and the population of low-performing students with no specific learning disability. Although the majority of studies dealt with procedural knowledge, many students with learning disabilities in mathematics struggle with what are considered basic mathematical procedures. This, in turn, limits their ability to solve more-complex problem types in which basic procedures are embedded.

Student Think-Alouds

Studies showed that when faced with multistep problems, students frequently attempted to solve the problems by randomly combining numbers instead of implementing a solution strategy step by step. The process of encouraging students to verbalize their thinking—by talking, writing, or drawing the steps they used in solving a problem—was consistently effective. In part, this procedure may be effective because the impulsive approach to solving problems taken by many students with mathematics difficulties was addressed. Results of these students were quite impressive, with an average effect size of 0.98, which is very large. In one set of studies, teachers provided numerous explicit models of how to solve a problem or a type of problem. They had students practice verbalizing a solution. A good deal of time went into how to solve, for example, the different types of subtraction problems by using part-whole relationships. This verbalization appeared to help anchor the students both behaviorally and mathematically.

Peer-Assisted Learning Activities and Formative Assessment Data

The role of peer-assisted learning and ongoing formative assessment data will be discussed in forthcoming NCTM research briefs. Results are quite promising for using peer-assisted learning with low-performing students but much more uncertain for special education students in the general classroom. The use of ongoing formative assessment data invariably improved mathematics achievement of students with mathematics disability.

Conclusion

In summary, the relatively small body of instructional research suggests several important teaching practices. For low-achieving students, the use of structured peer-assisted learning activities, along with systematic and explicit instruction and formative data furnished both to the teacher and to the students, appears to be most important. For special education students, explicit, systematic instruction that involves extensive use of visual representations appears to be crucial. In many situations with special education students, it is often advantageous for students to be encouraged to think aloud while they work, perhaps by sharing their thinking with a peer. These approaches also seem to inhibit those students who try too quickly and impulsively to solve problems without devoting adequate attention to thinking about what mathematical concepts and principles are required for the solution. Instruction should ideally be in a small group of no more than six and (*a*) address skills that are necessary for the unit at hand, (*b*) be quite explicit and systematic, and (*c*) require the student to think aloud as she or he solves problems or uses graphic representation to work through problem-solving options. Finally, it should balance work on basic whole-number or rational-number operations (depending on grade level) with strategies for solving problems that are more complex. These criteria should be considered in evaluating intervention programs for working with these types of students.

Adapted from a research analysis written by Russell Gersten and Benjamin S. Clarke.

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