RtI in Math for Elementary and Middle Schools

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Recommendation 1

Screen all students to identify those at risk for potential mathematics difficulites and provide interventions to students identified as at risk.

Level of evidence: Moderate

Recommendation 1

As a district or school sets up a screening system, have a team evaluate potential screening measures. The team should select measures that are efficient and reasonably reliable and that demonstrate predictive validity. Screening should occur in the beginning and the middle of the year.

Recommendation 2

Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 – 8. These materials should be selected by committee.

Level of evidence: Low (based on professional opinion)

Why this recommendation?

- Professional opinion based on consensus documents: NCTM Focal Points, National Math Panel
- Cover fewer topics in more depth, with coherence
- Numbers and operations are critically important to learning further math

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What?

- counting
- the base 10 place value system
- addition, subtraction, multiplication, division
  - what these operations mean
  - what kinds of problems they solve
  - basic facts — including relationships and strategies
  - algorithms — including the reasoning that underlies them

Recommendation 2
Focus on whole numbers in grades K – 5

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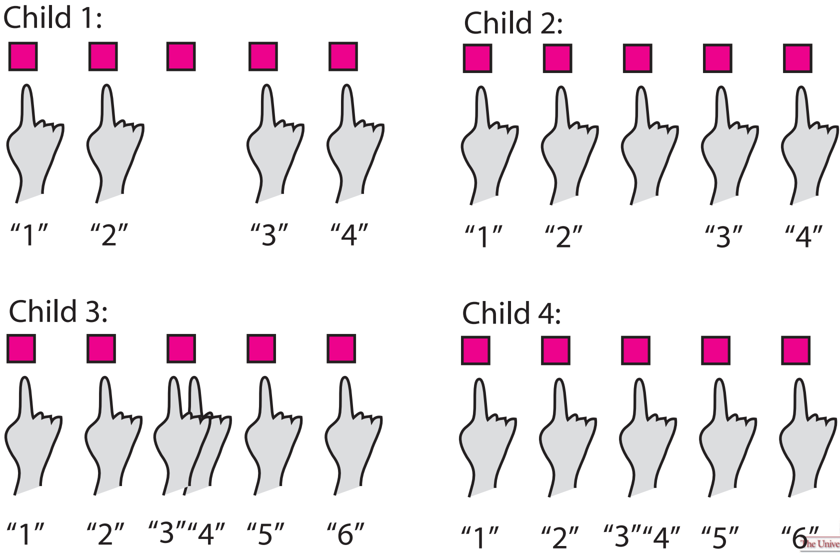
Counting

If a child can correctly say the first five counting numbers,

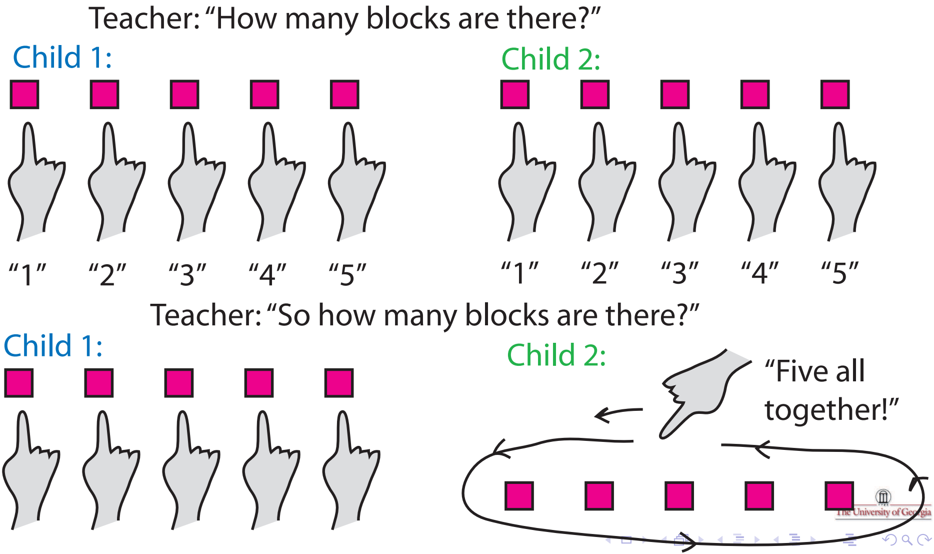
“one, two, three, four, five,”

will the child necessarily be able to determine how many blocks there are in this collection?

Counting

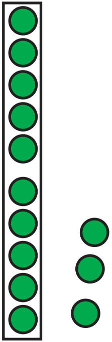


Counting


Teacher: "How many blocks are there?"
Child 1:
"1" "2" "3" "4" "5"
Child 2:
"1" "2" "3" "4" "5"
Teacher: "So how many blocks are there?"
Child 1:
Child 2:
"Five all together!"

Subtracting by decomposing 10

13 - 9



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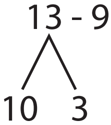

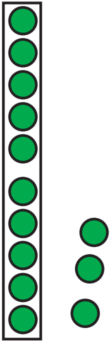

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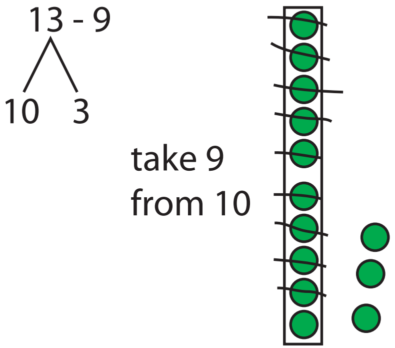

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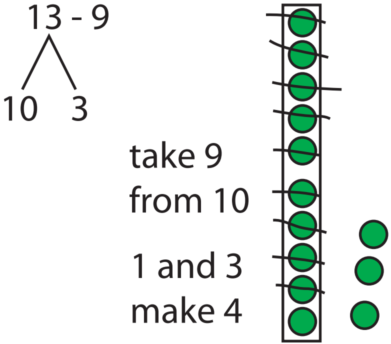
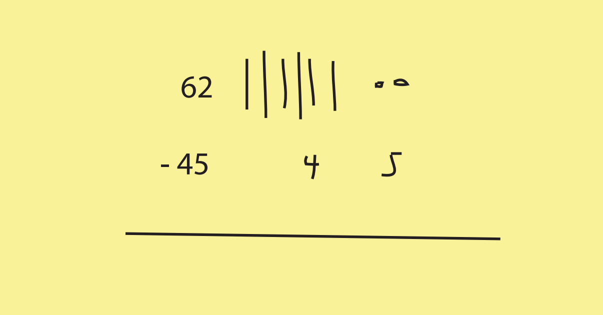


The common subtraction algorithm

```
  62
- 45
______
```

The common subtraction algorithm



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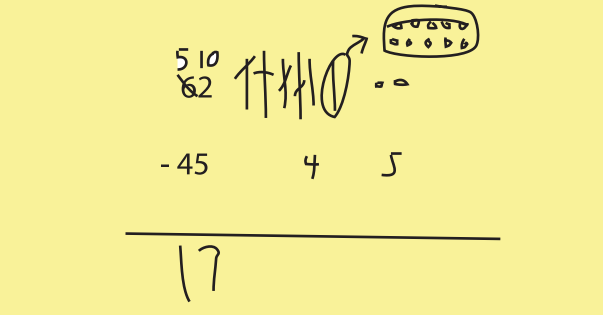
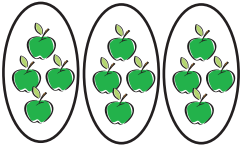
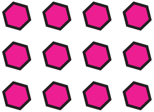
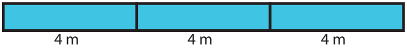


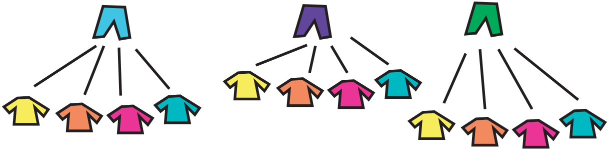
Definition of multiplication: $A \times B$ means the total in A groups of B (for non-negative A and B)





3×4





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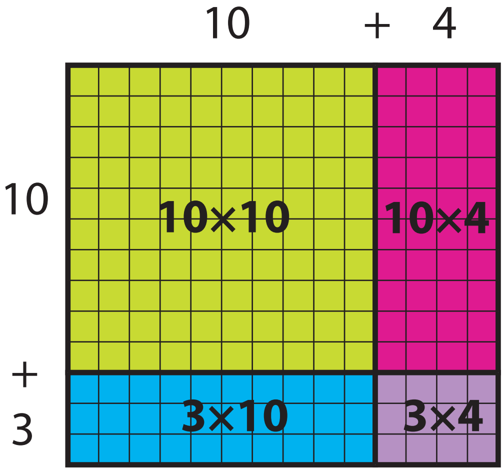
The Multiplication Algorithm

the “partial products” algorithm is a step toward the condensed standard algorithm



$$\begin{array}{r} 14 \\ \times 13 \\ \hline 12 \\ 30 \\ 40 \\ 100 \\ \hline 182 \end{array}$$

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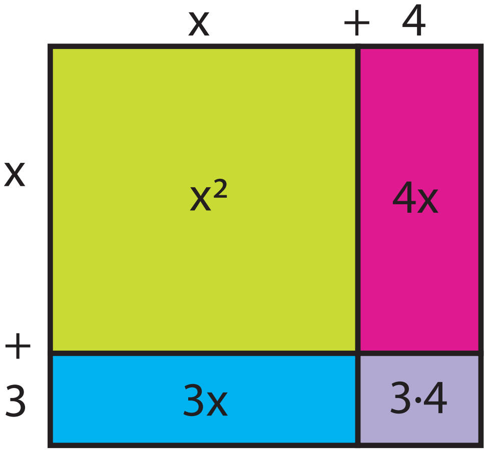
The Multiplication Algorithm

the same reasoning (applying the distributive property) is used in algebra



$$(x+3)(x+4) = x^2 + 3x + 4x + 3 \cdot 4$$

What?

- continuing emphasis on the base 10 place value system, extended to decimals — representing decimals as lengths, on number lines
- fractions — what they mean, representing them with fraction strips and on number lines
- continuing emphasis on addition, subtraction, multiplication, division
  - how the meaning extends to fractions and decimals and what kinds of problems these operations solve
  - why the procedures make sense
- ratio and proportion, percent

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Recommendation 2
Focus on rational numbers in grades 4 – 8

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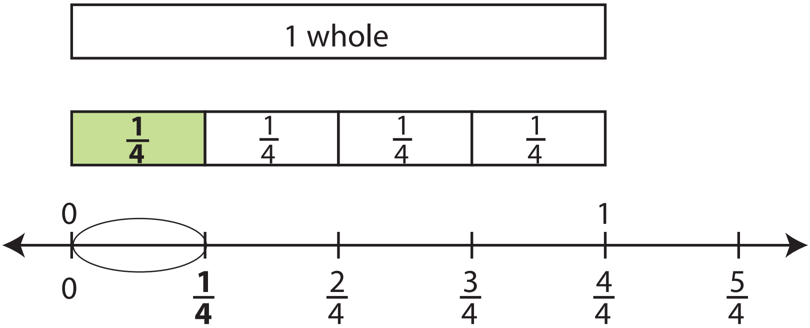
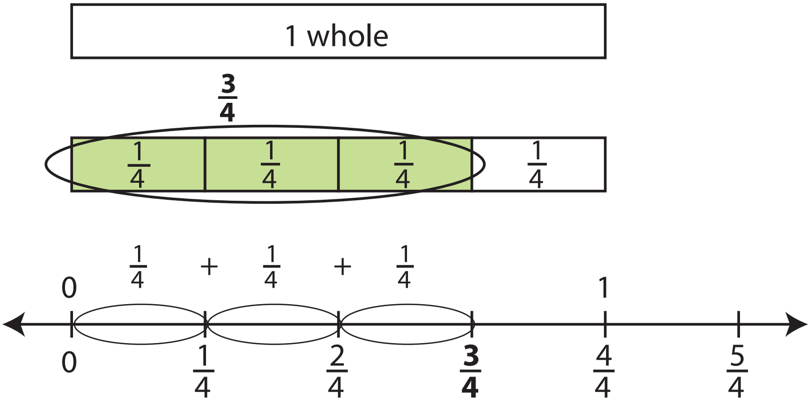
Representing fractions
Unit fractions first



Representing fractions



Fraction multiplication

Darrel has $\frac{1}{3}$ of a package of cheese left. He cuts off $\frac{1}{4}$ of it. What fraction of the package of cheese did he cut off?

"$\frac{1}{4}$ of $\frac{1}{3}$" is $\frac{1}{4} \times \frac{1}{3}$

just as

"4 of 3" is 4×3

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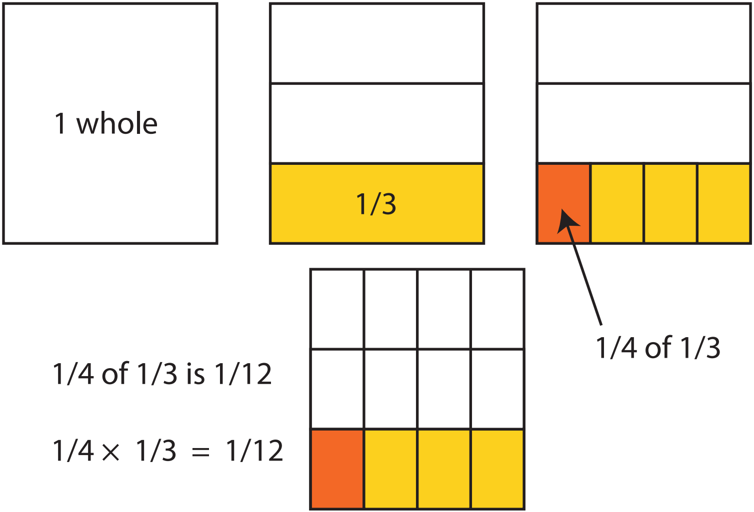
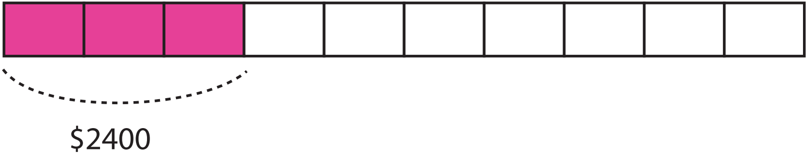


Percent problems

30% of the budget is \$2400. What is the full budget?



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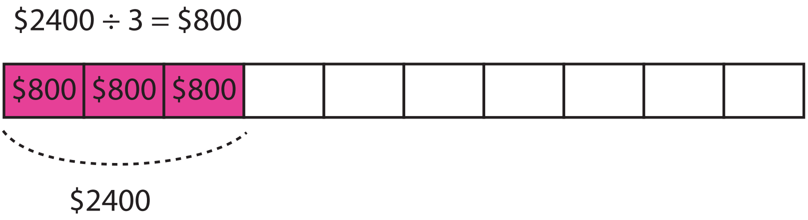

$$
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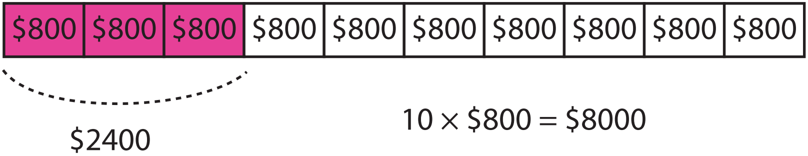
$\$2400 \div 3 = \$800$



$10 \times \$800 = \$8000$

$$\begin{array}{rcl} 30\% & \longrightarrow & \$2400 \\ 10\% & \longrightarrow & \$800 \\ 100\% & \longrightarrow & \$8000 \end{array}$$

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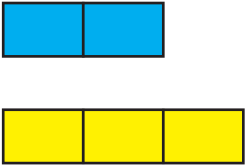
Ratio problems

Blue and yellow paint are mixed in a ratio of 2 to 3 to make green paint. How many pails of blue paint and how many pails of yellow paint will you need to make 30 pails of green paint?



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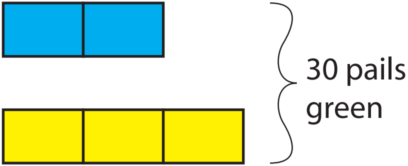

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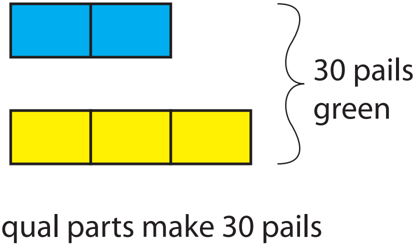

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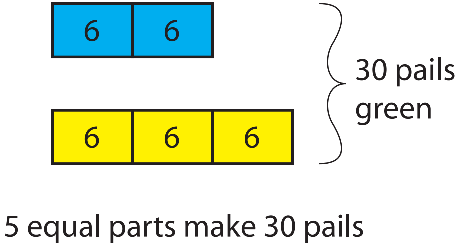

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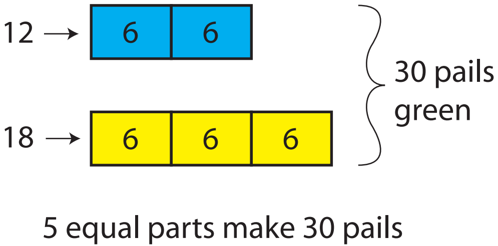

Recommendation 3

Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

Level of evidence: Strong

Discussing mathematical reasoning

Provide students with opportunities to solve problems in a group and communicate problem-solving strategies.

Discussing mathematical reasoning

"Professional development should provide participants with in-depth knowledge of the mathematics content in the intervention, including the mathematical reasoning underlying procedures, formulas, and problem-solving methods."

Recommendation 4

Interventions should include instruction on solving word problems that is based on common underlying structures.

Level of evidence: Strong

Recommendation 4

"Simple word problems give meaning to mathematical operations such as subtraction or multiplication. When students are taught the underlying structure of a word problem, they not only have greater success in problem solving but can also gain insight into the deeper mathematical ideas in word problems."

Why focus on structure of word problems?

What about teaching students to use key words?

- Good idea?
- Potential problems?

Focus on structure

Problem: After Amanda got 14 more buttons she had 52 buttons in all. How many buttons did Amanda have before she got more?

“got more” may lead students to add 14 to 52

Focus on structure

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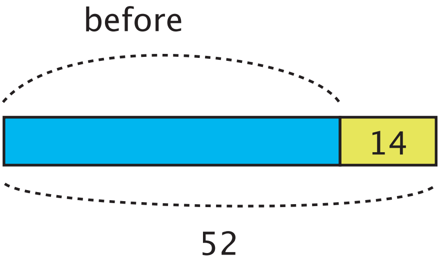
Focus on structure

A change problem:



Let B be the number of buttons at first, then

$$B + 14 = 52$$

Focus on structure

A multiplicative comparison problem:

Problem: Shauntay collected 5 times as many cans as Carla. If Shauntay collected 60 cans, how many did Carla collect?

Focus on structure

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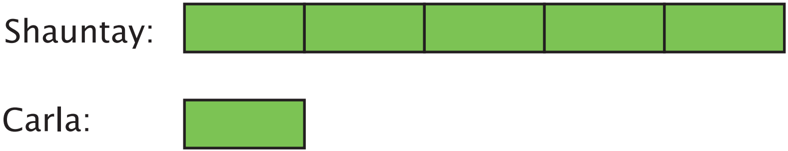

Recommendation 5

Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.

Level of evidence: Moderate

Recommendation 5

Use visual representations such as number paths, number lines, arrays, strip diagrams, other simple drawings or pictorial representations to scaffold learning and pave the way for understanding the abstract version of the representation.

Visual representations

Some thoughts on appropriate use of visual representations:

- Drawings should be *simple* and not involve distracting details.
- Link manipulatives and visual representations to standard mathematical notation.
- The goal is to get to standard mathematical notation and procedures, but to get there *with understanding*.

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Recommendation 6

Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.

Level of evidence: Moderate

Recommendation 6

Quick retrieval of basic arithmetic facts is critical for success in mathematics.

Recommendation 6

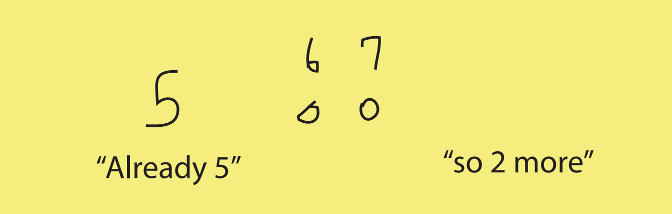
For students in kindergarten through grade 2, explicitly teach strategies for efficient counting to improve the retrieval of mathematics facts.

Counting on

A $5 + \square = 7$ problem:

Maya has 5 beads. She needs 7. How many more beads does Maya need?

Children can solve this by counting on from 5:



Counting on to subtract

A $7 - 5 = \square$ problem:

There were 7 nuts. Then a mouse ate 5. How many nuts are left?

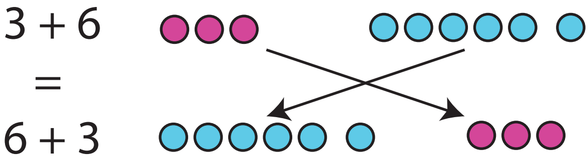
Children can also solve this by counting on from 5:

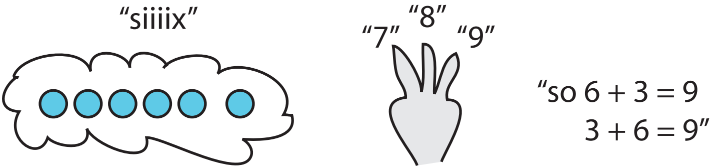
This method links subtraction and addition:

$$7 - 5 = \square \quad \leftrightarrow \quad 5 + \square = 7$$

Count on from larger

$3 + 6 = ?$



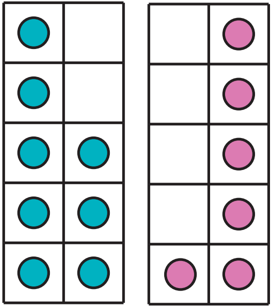


Make-a-ten methods

emphasize the base ten place value system

The **make-a-ten method** relies on breaking numbers apart and implicitly uses the associative property of addition:

$$8 + 6$$



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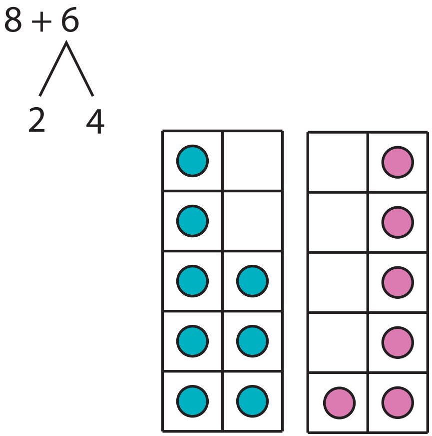

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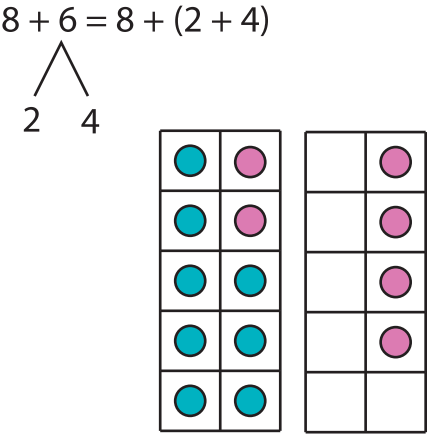
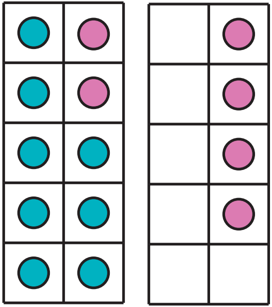


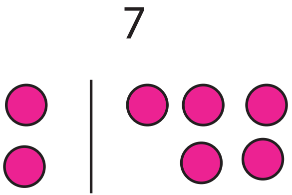
Make-a-ten methods
emphasize the base ten place value system

The **make-a-ten method** relies on breaking numbers apart and implicitly uses the associative property of addition:

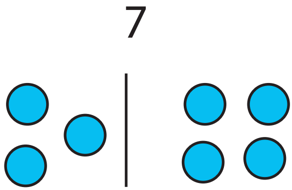
$$8 + 6 = 8 + (2 + 4) = (8 + 2) + 4 = 14$$

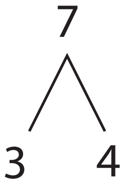


Break numbers apart
A prerequisite for make-a-ten methods



$$7 = 2 + 5$$





$$7 = 3 + 4$$

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Recommendation 6

Teach students in grades 2 through 8 how to use their knowledge of properties, such as the commutative, associative, and distributive laws, to derive facts in their heads.

The commutative property of multiplication

For all numbers A, B,

$$A \times B = B \times A$$

For example, $3 \times 5 = 5 \times 3$.

Cuts down the memorization load of the basic multiplication facts!

Is it obvious why this property is true?

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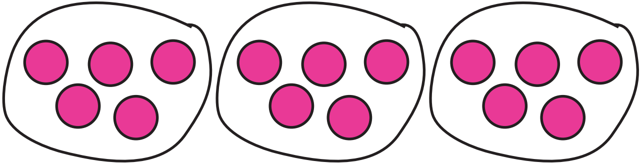
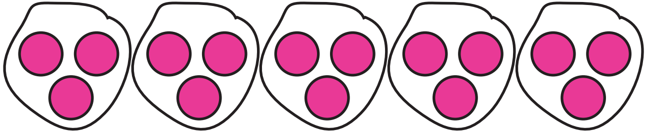
The commutative property of multiplication

It's not obvious that the commutative property is true!

3×5



5×3

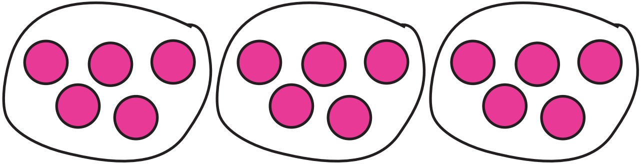


So why *is* it the case that $A \times B$ is equal to $B \times A$?

The commutative property of multiplication

It's not obvious that the commutative property is true!

3×5



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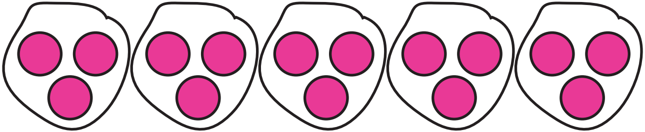

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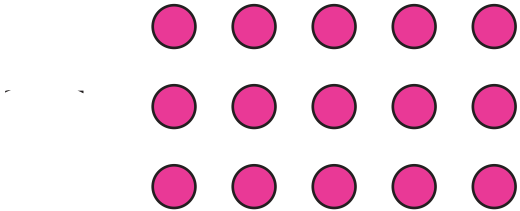
The commutative property of multiplication



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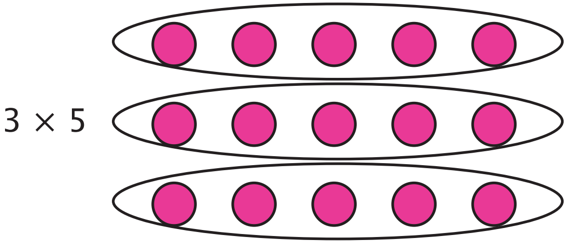

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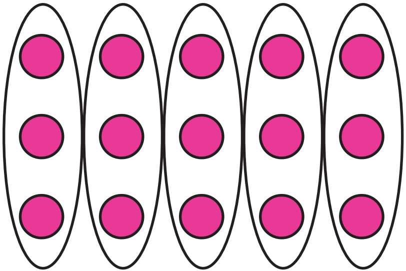

5×3

The distributive property

$$4\times7 = 4\times5 + 4\times2$$
$$4\times(5+2) = 4\times5 + 4\times2$$

The University of Georgia

Connecting basic multiplication facts

via properties of arithmetic

Reasoning about relationships among basic facts is important not only for scaffolding student learning for automaticity but also for understanding the multiplication algorithm, algebra, and area and volume calculations.

$6\times 7 = 5\times 7 + 1\times 7$ $\quad$ $6\times 7 = 2\times(3\times 7)$ $\quad$ $6\times 7 = 6\times 5 + 6\times 2$

Relationships among basic facts

Studying basic facts by examining and using relationships among facts allows for:

- the kind of "algebraic reasoning" of taking apart, working with pieces, and putting back together that is important throughout math
- thinking about the meaning of operations
- developing number sense

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Recommendation 7

Monitor the progress of students receiving supplemental instruction and other students who are at risk.

Level of evidence: Low

Recommendation 7

Use progress monitoring data to regroup students within tiers so that the small groups used in tier 2 interventions are as homogeneous as possible.
Consider grouping students across classes.

Recommendation 8

Include motivational strategies in tier 2 and tier 3 interventions.

Level of evidence: Low (not much research is available)

Recommendation 8

Promote student effort — engagement-contingent rewards

Promote persistence — completion-contingent rewards

Promote achievement — performance-contingent rewards

Time for discussion

Your thoughts and advice to each other on:

- screening
- progress monitoring
- explicit, systematic instruction
- motivational strategies

- math content to emphasize
- teaching the structure of word problems
- using visual representations
- helping students learn the basic arithmetic facts